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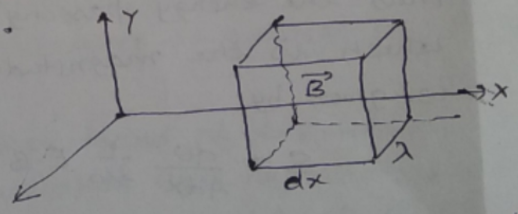
Q Define Poynting vector. Derive an expression and explain its physical signifance for Plane e.m wave. Also calculate Poynting theorem for the flow of energy in an electromagnetic field. Find out the energy density and momentum related to Poynting vector?

Ans Poynting vector :→

Like any other travelling wave, an e.m. wave transports energy from point to point. The rate of energy flow per unit area ⊥ to the direction of propagation of plane e.m wave can be described by a propagation vector $\vec{S}$, known as Poynting vector and is defeined as $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$

$\vec{S}$ is expressed in Wm^{-2}, Here $\vec{E}$ = Electric field and $\vec{B}$ = magnetic field. The direction of $\vec{S}$ is ⊥ to the plane containing $\vec{E}$ & $\vec{B}$ is the direction of propagation of wave.

Suppose a plane e.m wave is propagating in the x-direction. Let us consider a box of length dx and cross sectional area A which lies in the plane of $\vec{E} \times \vec{B}$. The box is fixed w.r. to the axes while the wave moves through it.



We know that the energy density in an electric fields is $\frac{1}{2}\epsilon_0 E^2$ and that in a magnetic field is $\frac{1}{2\mu_0} B^2$ in the free space. Therefore at any instant the electromagnetic energy in the box is

$$dU = \left\{ \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0} B^2 \right\} A \cdot dx \quad \text{—— (1)}$$

Here $A \cdot dx$ is the volume of the box and $\vec{E}$ and $\vec{B}$ are the instantaneous value of the field vectors in the box.

$\vec{E}$ and $\vec{B}$ are related as

$$\vec{E} = c\vec{B}$$

from eq^n (1)

$$dU = \left[\frac{1}{2}\epsilon_0 E(cB) + \frac{1}{2\mu_0} B(E/c) \right] A \cdot dx$$

$$= \frac{EB}{2\mu_0 c} (\mu_0 \epsilon_0 c^2 + 1) A \cdot dx$$

But $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ So that $\mu_0 \epsilon_0 c^2 = 1$

$$\therefore dU = \frac{EB}{\mu_0 c} A \cdot dx$$

The instantaneous energy-density in e.m. waves is

$$U = \frac{dU}{A dx} = \frac{EB}{\mu_0 c} \quad \text{—— (II)}$$

The e.m. wave with velocity c in the x-direction. If dt be the time taken by the wave to cover the length dx of the box, then $dx = c\,dt$

$$\therefore dU = \frac{EB}{\mu_0} A \cdot dt$$

The energy dU now in the box will pass through the right face of the box in a time dt. Thus the energy passing per unit area per unit time which is the magnitude S of the poynting vector is given by —— (III)

$$S = \frac{dU}{A dx} \frac{1}{\mu_0} E \cdot B$$

'S' is known as energy flux or Energy density

For the ~~gerene~~ general case in vector notation it would be

$$\vec{S} = \frac{1}{\mu_0} (\vec{E} \times \vec{B})$$

This relation gives the instantaneous rate of energy flow per unit area

Momentum of e.m. wave :→

Maxwell also predicted that e.m. waves transport linear momentum in the direction of propagation. It means that they exert a pressure when falling on the surface of body.

Suppose that a plane e.m. wave falls normally on a perfectly absorbing surface of area A for time 't'. If energy U is absorbed during this time, the momentum P delivered to the surface is given to Maxwell's prediction by

$$P = \frac{U}{c}$$

~~Here~~ If 'S' is the energy flux, then $U = SAt$

$$P = \frac{SAt}{c} \qquad \text{But } \frac{S}{c} = u \text{ (Energy density)}$$

$$\therefore P = uAt$$

From Newton's law, the $\langle F \rangle$ on the surface is equal to the average rate at which momentum is delivered to the Surface

$$F = \frac{P}{t} = uA$$

Dividing by Area A, we get the radiation pressure exerted on the surface

$$P_{rad} = \frac{F}{A} = u$$

Thus for the normal incidence the radiation pressure on a perfect absorber is equal to the energy density in the wave.

On the other hand, if the surface is perfect reflector, the radiation after reflection has a momentum equal to magnitude but opposite in direction to the incident radiation. The momentum delivered will therefore be twice than given above and the radiation pressure would be $P_{rad} = 2u$